Broadband Faraday Rotation Sections Using Three Ridges

B. M. Dillon and A. A. P. Gibson

Abstract—**Triply-ridged circular waveguide cross-sections are proposed for wideband Faraday rotation applications. Finite element calculations for both the cut-off planes and the phase constants are presented to describe modal behavior. A comparison with the more conventional quadruply-ridged section indicates that there is a significantly increased single-moded bandwidth in the case of the triply-ridged section. In both cases the ridges reduce phase constant dispersion in the dominant pair of degenerate modes to provide broadband Faraday rotation.**

I. Introduction

NON-RECIPROCAL COMPONENTS at microwave and millimeter frequencies are often realized using Faraday rotation in ferrite-loaded waveguides. The classic geometry for Faraday rotation is a centrally-positioned ferrite rod in a circular waveguide. When an axial magnetic field is applied the degeneracy of the counter-rotating circularly-polarized modes is split so that linear-polarized waves rotate as they propagate. Broadband devices have previously been constructed by modifying the geometry to reduce the cut-off wavenumber of the dominant modes. One approach is to use dielectric loading but this has the disadvantage of reducing the cut-off numbers of all the modes [1], so that the useful single-mode bandwidth of the Faraday rotator is also reduced. An alternative method is to introduce four identical ridges into the circular waveguide [2]–[4]. Unlike dielectric loading, ridges do not affect the cut-off numbers of all the modes in the same way. In general the cut-off numbers of the limit-TE modes reduce whereas those of the limit-TM are observed to increase. One difficulty with this arrangement is the tendency of one of the higher order TE modes to have a greatly reduced cut-off wavenumber close to that of the dominant mode. This reduces the single moded bandwidth and degrades the performance of the rotator. To avoid this problem an alternative triply-ridged circular waveguide cross-section is proposed here. Three magnetic-wall ridges have previously been introduced to tune planar resonators [5].

Since ridged waveguide cross-sections are no longer axi-symmetric, numerical methods such as the finite element method must be used to analyze their characteristics. The calculated results show that the three-fold symmetry of the triply-ridged case is sufficient to support pairs of degenerate modes necessary for Faraday rotation. In addition, the effect of the ridges on the dominant circularly-polarized modes is similar to that in the quadruply-ridged waveguide. However in the triply-ridged case the use of fewer ridges increases the separation between the modes in the cut-off plane giving an increased single-mode bandwidth.

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The authors are with the Department of Electrical Engineering and Electronics, UMIST, Manchester, M60 1QD, United Kingdom.

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II. Finite Element Method

Two different finite element formulations have been implemented: one calculates the wavenumber for a specified phase constant and the other calculates the propagation constant at a specified frequency. The results from both formulations have been validated against known solutions [6], [7]. Results described here were obtained using meshes of about 30 mixed-order covariant-projection elements defined over a quarter or a third of the waveguide cross-section depending upon the symmetry. The use of symmetry planes in finite element calculations for anisotropic waveguides has recently been described [8].

III. Faraday-Rotation Calculations

The geometry of a triply-ridged Faraday rotation section is shown schematically in Fig. 1. The Faraday rotation obtained from this cross-section over a fixed length L was calculated using the standard formula

$$\theta(f) = \left(\frac{\beta_+(f) - \beta_-(f)}{2}\right)L$$

where $\beta_\pm(f)$ are the phase constants of the circularly-polarized dominant $HE_{\pm 11}$ modes which have been calculated at a specified frequency f using the finite element method. The variation in the Faraday rotation θ with frequency is shown in Fig. 2. For comparison the results for the same magnetized ferrite rod with no ridges and with four ridges are also shown. The rotation increases rapidly with frequency for the axi-symmetric geometry with no ridges: $R = \theta$ (15 GHz) $/\theta$ (11 GHz) ≈ 4.0. For the quadruply-ridged geometry, the rotation stays more constant over the same bandwidth with $R \approx 1.18$. This broadband behavior, compared to the axi-symmetric case, is similar to that described elesewhere [2], [3]. The rotation for the triply-ridged case shows similar behavior with $R \approx 1.23$. There is a slight reduction in rotation per unit length with the triply-ridged geometry compared to the quadruply-ridged case. One explanaton for this is that decreasing the number of ridges reduces the proportion of the field in the ferrite medium.

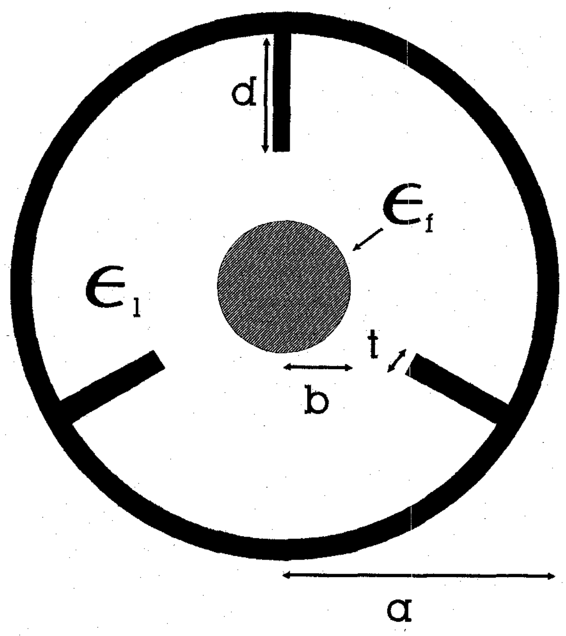

Fig. 1. Geometry of an empty triply-ridged waveguide with ferrite rod.

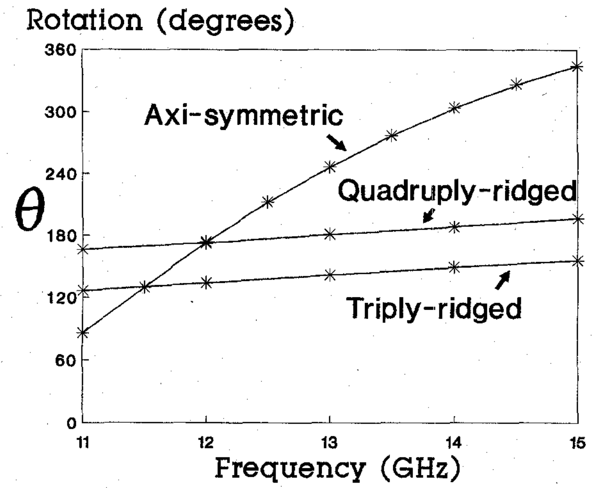

Fig. 2. Faraday rotation for $L = 20$ mm in axisymmetric, quadruply-ridged and triply-ridged ferrite waveguides: $a = 11.912$ mm, $b/a = 0.2$ $d/a = 0.80$, $t/a = 0.07$, $\epsilon_f = 13.0$, $\epsilon_1 = 1.0$, $M_s = 2650$ Gauss and $H_{dc} = 400$ Oe.

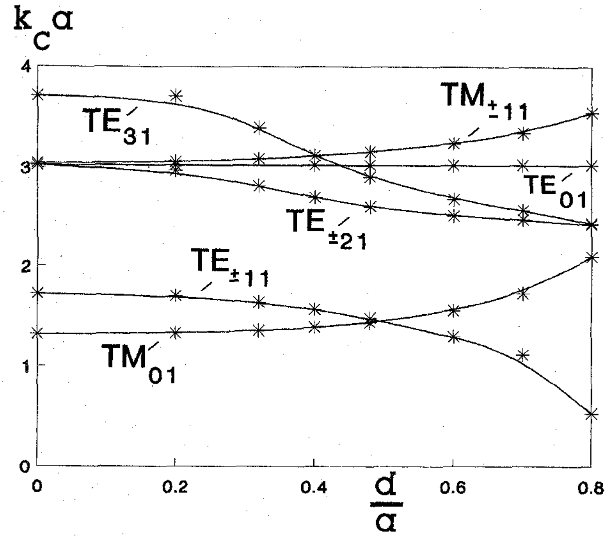

Fig. 3. Variation in cut-off wavenumbers with ridge length for triply-ridged waveguide with unmagnetized ferrite: $a = 11.912$ mm, $b/a = 0.2$, $t/a = 0.0$, $\epsilon_f = 13.0$, $\epsilon_1 = 1.0$.

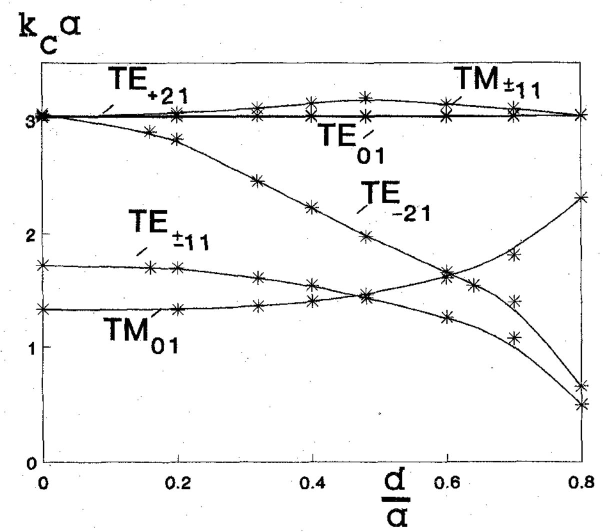

Fig. 4. Variation in cut-off wavenumbers with ridge length for quadruply-ridged waveguide with unmagnetized ferrite: $a = 11.912$ mm, $b/a = 0.2$, $t/a = 0.0$, $\epsilon_f = 13.0$, $\epsilon_1 = 1.0$.

IV. Cut-Off Plane Calculations

The broadband behavior in the ridged Faraday rotation sections described above can be understood by studying the effect of the ridges on the cut-off wavenumber of the dominant mode. For the geometry illustrated in Fig. 1, the finite-element calculated cut-off frequency of the limit-$TE_{\pm 1,1}(HE_{\pm 1,1})$ is 1.77 GHz and that of the identical quadruply-ridged section is similar at 1.86 GHz. These are significantly less than the 6.88 GHz cut-off frequency of the axisymmetric case and so the ridged cross-sections are less dispersive.

To examine the single-mode bandwidth of the Faraday rotators, the cut-off frequencies of the higher order modes must also be investigated. Finite element calculations for the variation in the cut-off wavenumber with ridge length for a triply-ridged ferrite-loaded waveguide are shown in Fig. 3. Similar calculations for an identical geometry but with four ridges are shown in Fig. 4. For convenience the ridge thickness is taken to be zero in both cases. In practice, the thickness has a less significant effect on the cut-offs compared with the ridge length [2]. In general, introducing ridges into a waveguide tends to increase the cut-offs of the limit-$TM_{nm}(EH_{nm})$ modes and decrease the cut-offs of the limit-$TE_{nm}(HE_{nm})$ modes. Waldron [1] has proven that for an axi-symmetric geometry only the cut-offs of the limit-TM modes are affected by the magnetization of the ferrite. This was also found to be the case for the ridged circular cross-section. Calculation shown in Figs. 2 and 3 are for an unmagnetized rod; with an applied bias field the cut-offs of limit-TM modes only would change by increasing slightly.

The results in Figs. 2 and 3 show that both ridged cross-sections support a pair of degenerate $TE_{\pm 11}$ modes which can be used to give Faraday rotation. The cut-offs of these modes decrease in both cases with increasing ridge length. The most important difference between the two cut-off planes is that for quadruply-ridged case the $TE_{\pm 21}$ modes are split by the ridges. The cut-off of one of these modes decreases sharply with increasing ridge length and for fairly long ridges, it is close in cut-off to the $TE_{\pm 11}$ pair. In the triply-ridged case, the higher order $TE_{\pm 21}$ modes like the $TE_{\pm 11}$ modes are not split. Although the $TE_{\pm 31}$ modes do split, the cut-off of the lower mode is still well above the cut-off of the dominant $TE_{\pm 11}$ modes. Thus with long ridges, the triply-ridged ferrite waveguide is single-moded over a wider frequency bandwidth.

V. Conclusion

The finite element results described here indicate that a triply-ridged waveguide cross-section can be used in broad-band Faraday rotators. An important advantage of the triply-ridged cross-section over conventional methods is that it has a significantly larger single-method moded bandwidth.

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